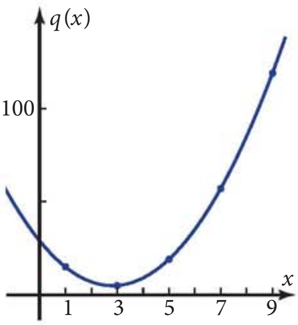
The Constant-Second-Differences Pattern of Quadratic Functions



Quadratic function

	x	$f(x)$		
	1	15		
+2			−10	
	3	5		+24
+2			+14	
	5	19		+24
+2			+38	
	7	57		+24
+2			+62	
	9	119		

Constant second differences

First differences

Constant second differences

Figure 2-3d

Figure 2-3d shows the graph of the quadratic function $q(x) = 3x^2 - 17x + 29$. An extension of the add–add property for linear functions applies to quadratics. For equally spaced x-values, the *differences* between the corresponding y-values are equally spaced. Thus the differences between these differences (the *second differences*) are constant. This constant is equal to $2ad^2$, twice the coefficient of the quadratic term times the square of the difference between the x-values.

These four properties are summarized in the box.

PROPERTIES: Patterns for Function Values

Add–Add Property of Linear Functions

If f is a linear function, adding a constant to x results in adding a constant to the corresponding $f(x)$-value. That is,

$$\text{if } f(x) = ax + b \text{ and } x_2 = c + x_1, \text{ then } f(x_2) = ac + f(x_1)$$

Add–Multiply Property of Exponential Functions

If f is an exponential function, adding a constant to x results in multiplying the corresponding $f(x)$-value by a constant. That is,

$$\text{if } f(x) = ab^x \text{ and } x_2 = c + x_1, \text{ then } f(x_2) = b^c \cdot f(x_1)$$

Multiply–Multiply Property of Power Functions

If f is a power function, multiplying x by a constant results in multiplying the corresponding $f(x)$-value by a constant. That is,

$$\text{if } f(x) = ax^b \text{ and } x_2 = cx_1, \text{ then } f(x_2) = c^b \cdot f(x_1)$$

Constant-Second-Differences Property of Quadratic Functions

If f is a quadratic function, $f(x) = ax^2 + bx + c$, and the x-values are spaced d units apart, then the second differences between the $f(x)$-values are constant and equal to $2ad^2$.